

Name: _____

GCSE (1 – 9)

Circle Theorems

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

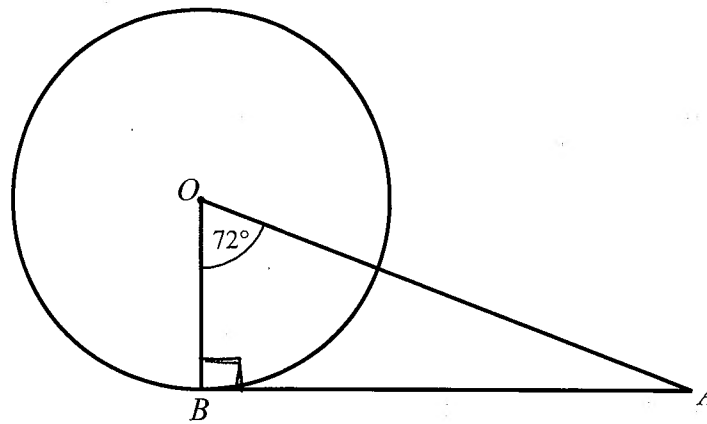
Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1



B is a point on the circumference of a circle, centre O .
 AB is a tangent to the circle.

Angle $BOA = 72^\circ$

Work out the size of angle BAO .
 You must show all your working.

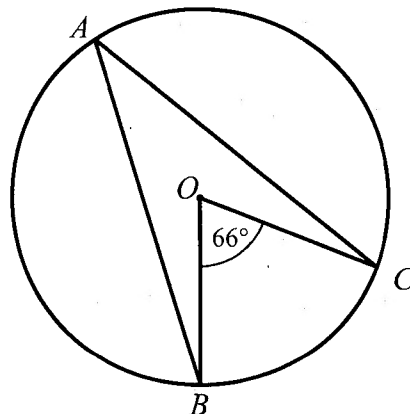
$$OBA = 90^\circ \quad \text{Tangent meets radius at } 90^\circ$$

$$\begin{aligned} BAO &= 180 - 90 - 72 & \text{Angles in a triangle} \\ &= \underline{\underline{18^\circ}} & \text{add to } 180^\circ \end{aligned}$$

18

(Total for Question 1 is 2 marks)

2



A , B , C and D are points on the circumference of a circle.

Angle $BOC = 66^\circ$

(i) Find the size of angle BAC .

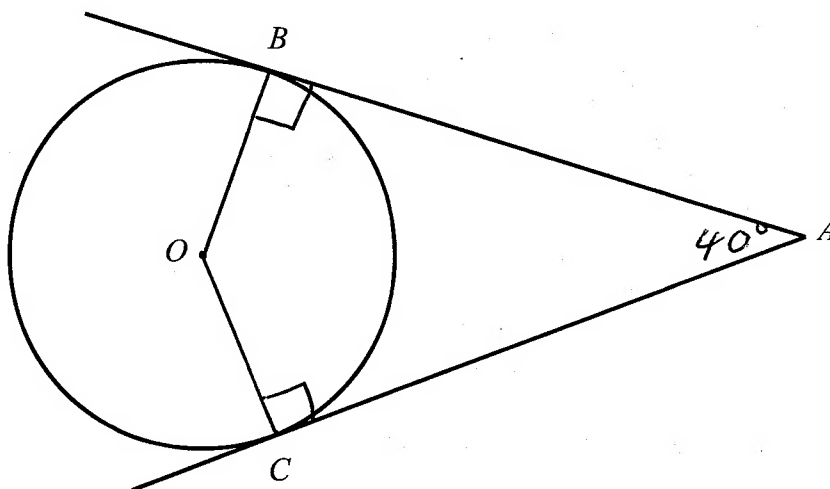
(ii) Give a reason for your answer.

33

The angle at the circumference is half the angle at the centre

(Total for Question 2 is 2 marks)

3



B and C are points on a circle, centre O .
 AB and AC are tangents to the circle.

Angle $BAC = 40^\circ$

Work out the size of angle BOC .
 You must show all your working.

OBA and $OCA = 90^\circ$
 Tangent meets radius at 90°

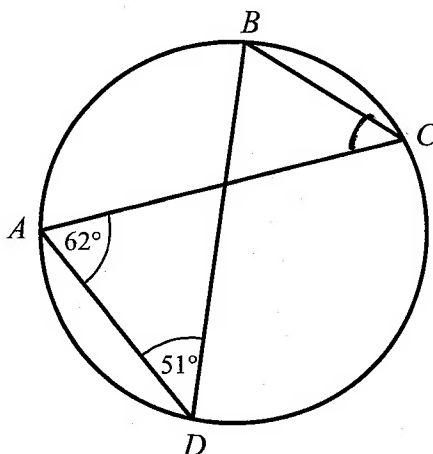
$$360 - 90 - 90 - 40 = 140^\circ$$

Angles in a quadrilateral
 add to 360°

140

(Total for Question 3 is 3 marks)

4



A , B , C and D are points on the circumference of a circle.

Angle $CAD = 62^\circ$

Angle $ADB = 51^\circ$

(i) Find the size of angle ACB .

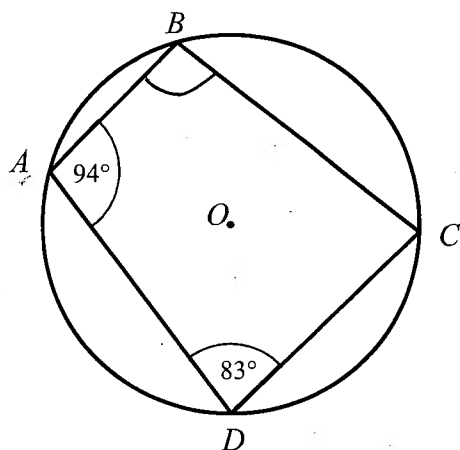
51

(ii) Give a reason for your answer.

Angles in the same segment are equal.

(Total for Question 4 is 2 marks)

5



A, B, C and D are points on the circumference of a circle.

Angle $BAD = 94^\circ$

Angle $ADC = 83^\circ$

$$180 - 83$$

(i) Find the size of angle ABC .

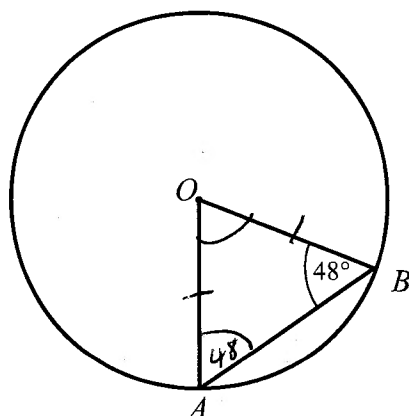
97

(ii) Give a reason for your answer.

opposite angles in a cyclic quadrilateral
add to 180°

(Total for Question 5 is 2 marks)

6



A and B are points on the circumference of a circle, centre O .

Angle $ABO = 48^\circ$

$OAB = 48^\circ$
Angles at base
of isosceles are
equal

(i) Find the size of angle AOB .

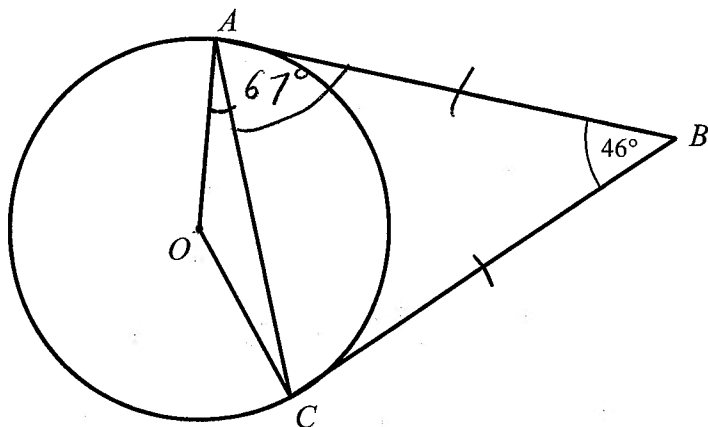
$$180 - 48 - 48$$

(ii) Give a reason for your answer.

84

Angles at the base of an isosceles triangle are
equal + angles in a triangle add to 180°

(Total for Question 6 is 2 marks)



A and C are points on the circumference of a circle, centre O .
 AB and BC are tangents to the circle.

Angle $ABC = 46^\circ$

Find the size of angle OAC .

Give reasons for each stage of your working.

ABC is an isosceles triangle, 2 tangents from the same point are equal

$$\angle CAB = \frac{180 - 46}{2} = \frac{134}{2} = 67^\circ$$

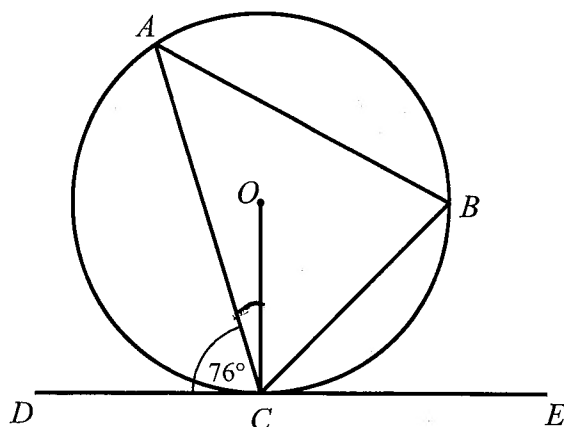
Angles at the base of an isosceles triangle are equal

$OAB = 90^\circ$ Tangent meets radius at 90°

$$\angle OAC = 90 - 67 = \underline{\underline{23^\circ}}$$

23°

(Total for Question 7 is 4 marks)



A and B are points on the circumference of a circle, centre O .
 DCE is a tangent to the circle.

Angle $ACD = 76^\circ$

- (a) Find the size of angle ACO .
 You must ~~show~~ all your working.
 Give reasons for

$$OCD = 90^\circ \quad \text{Tangent meets radius at } 90^\circ$$

$$90 - 76 = 14^\circ$$

14

 (2)

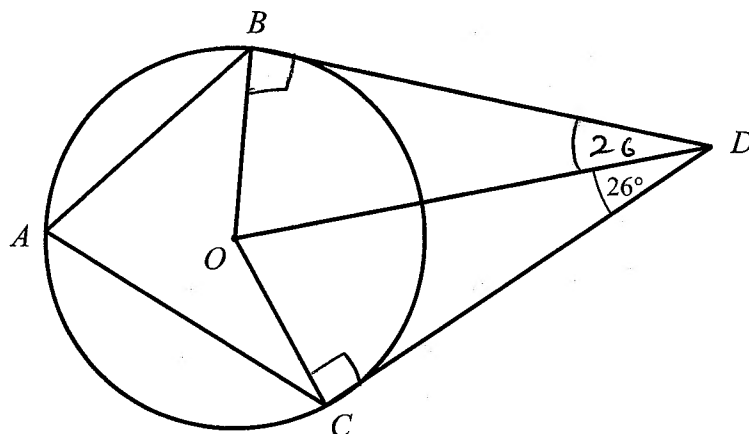
- (b) Find the size of angle ABC .
 You must ~~show~~ all your working.
 Give reasons for

$$ABC = 76^\circ \quad \text{Alternate segment theorem}$$

76

 (2)

(Total for Question 8 is 4 marks)



A , B and C are points on the circumference of a circle, centre O .
 BD and CD are tangents to the circle.

Angle $ODC = 26^\circ$

Find the size of angle BAC .

Give reasons for each stage of your working.

Triangle $BOD = \text{Triangle } DOC$ all sides are equal

OBD and $OCD = 90^\circ$ Tangent meets radius
at 90°

$$BOD \text{ and } COD = 180 - 90 - 26 \\ = 64^\circ$$

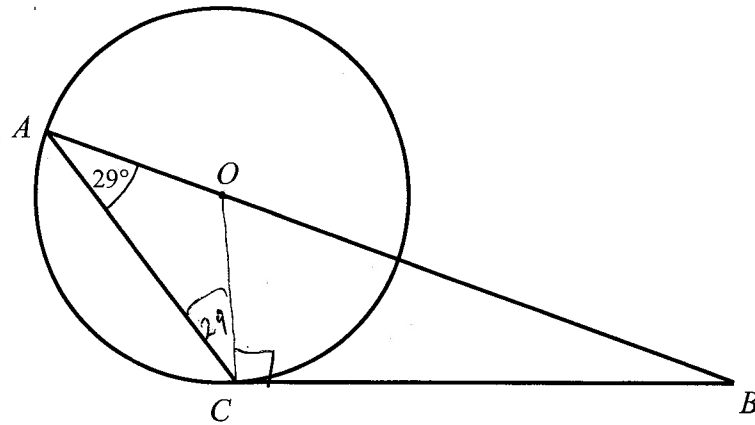
$$BOC = 2 \times 64 = 128^\circ$$

$$BAC = \frac{128}{2} = 64^\circ$$

Angle at circumference
is half the angle
at the centre

64°

(Total for Question 9 is 4 marks)



A and C are points on the circumference of a circle, centre O .
 BC is a tangent to the circle.

Angle $CAB = 29^\circ$

Find the size of angle ABC .
 You must show all your working.

$OCA = 29^\circ$ Angles at the base of
 an isosceles triangle
 are equal

$OCB = 90^\circ$ Tangent meets radius
 at 90°

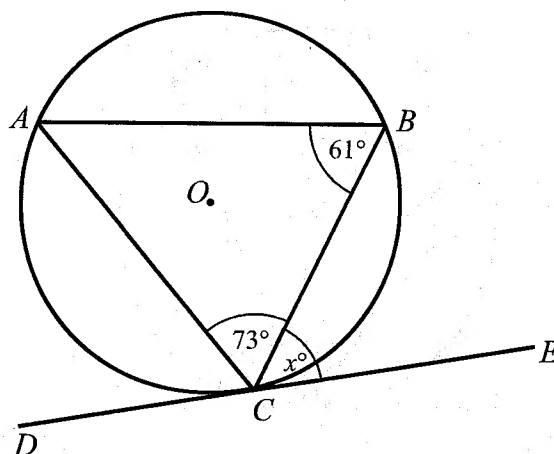
$$ABC = 180 - 90 - 29 - 29$$

$$= 32^\circ$$

Angles in a triangle
 add to 180°

32

(Total for Question 10 is 4 marks)



A , B and C are points on the circumference of a circle, centre O .
 DCE is a tangent to the circle.

Angle $ABC = 61^\circ$

Angle $ACB = 73^\circ$

Angle $BCE = x^\circ$

Find the value of x .

You must show all your working.

$$BAC = 180 - 73 - 61$$

$$= 46^\circ$$

Angles in a triangle
add to 180°

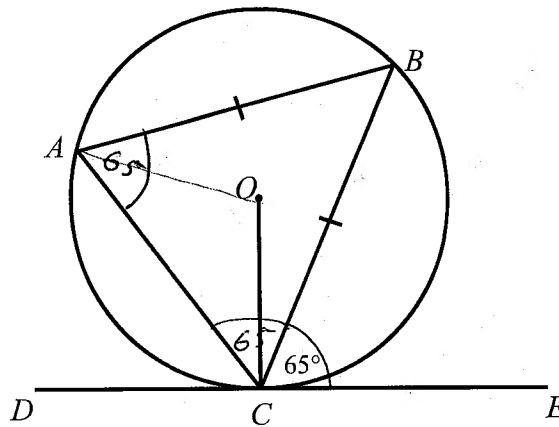
$$x = 46^\circ$$

Alternate segment theorem

46

°

(Total for Question 11 is 3 marks)



A , B and C are points on the circumference of a circle, centre O .
 DCE is a tangent to the circle.

$AB = BC$
 Angle $BCE = 65^\circ$

Find the size of angle AOC .
 You must show all your working.

$BAC = 65^\circ$ Alternate segment theorem

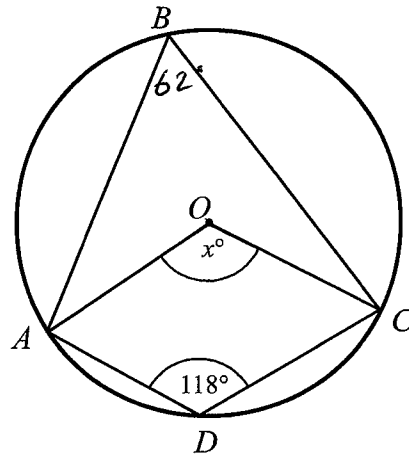
$ACB = 65^\circ$ Angles at the base of an isosceles triangle are equal

$ABC = 180 - 65 - 65$
 $= 50^\circ$ Angles in a triangle add to 180°

$AOC = 2 \times 50$
 $= 100^\circ$ Angle at centre is twice angle at circumference

100°

(Total for Question 12 is 4 marks)



A , B , C and D are points on the circumference of a circle, centre O .

Angle $ADC = 118^\circ$

Angle $AOC = x^\circ$

Work out the value of x .

You must show all your working.

$$\begin{aligned} \angle ABC &= 180 - 118 \\ &= 62^\circ \end{aligned}$$

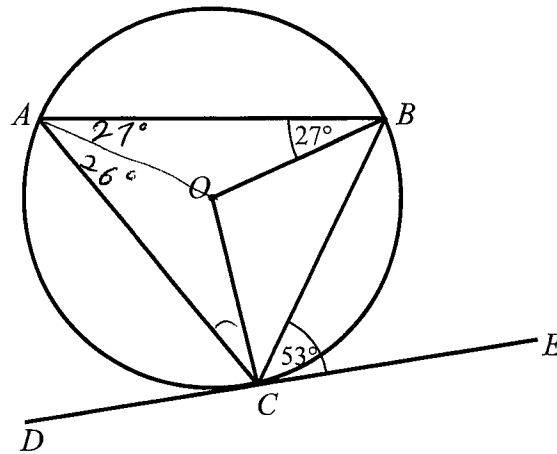
opposite angles in a cyclic quadrilateral add to 180°

$$\begin{aligned} \angle AOC &= 2 \times 62 \\ &= 124^\circ \end{aligned}$$

Angle at centre is twice angle at circumference

124

(Total for Question 13 is 3 marks)



A , B and C are points on the circumference of a circle, centre O .
 DCE is a tangent to the circle.

Angle $ABO = 27^\circ$

Angle $BCE = 53^\circ$

Find the size of angle ACO .

~~You must show all your working.~~

~~Give reasons for each stage of your working.~~

$BAO = 27^\circ$ Angles at base of isosceles triangle are equal

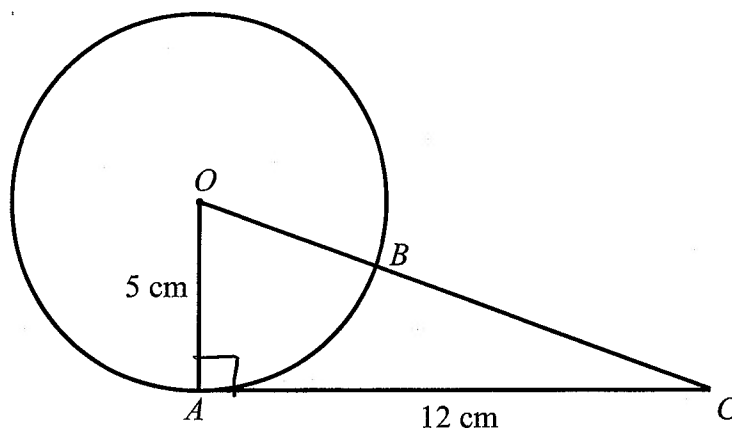
$BAC = 53^\circ$ Alternate segment theorem

$$OAC = 53 - 27 = 26^\circ$$

$ACO = 26^\circ$ Angles at base of isosceles triangle are equal

26°

(Total for Question 14 is 4 marks)



A and B is a point on the circumference of a circle, centre O .

AC is a tangent to the circle.

OBC is a straight line.

$OA = 5 \text{ cm}$

$AC = 12 \text{ cm}$

Find the length of BC .

You must show all your working.

$OAC = 90^\circ$ Tangent meets radius at 90°

$$5^2 + 12^2 = OC^2$$

$$169 = OC^2$$

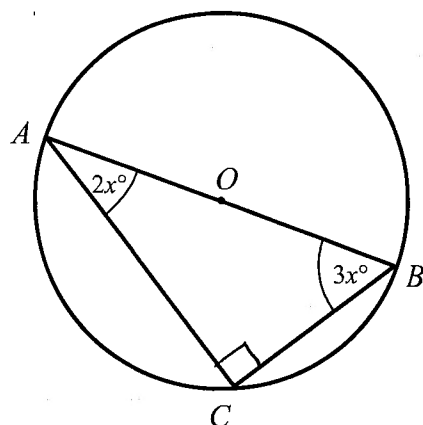
$$OC = \sqrt{169}$$

$$= 13 \text{ cm}$$

$$BC = 13 - 5 = 8 \text{ cm} \quad (\text{radius} = 5 \text{ cm})$$

8 cm

(Total for Question 15 is 4 marks)



A , B and C are points on the circumference of a circle, centre O .

Angle $CAB = 2x^\circ$

Angle $ABC = 3x^\circ$

Find the value of x .

You must show all your working.

$ACB = 90^\circ$ Angle in a semi circle
is 90°

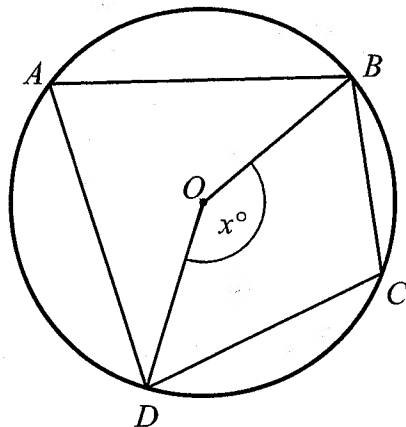
$$2x + 3x + 90 = 180$$

$$5x = 90$$

$$x = 18^\circ$$

$$x = \underline{\hspace{1cm}} / 8$$

(Total for Question 16 is 3 marks)



A , B , C and D are points on the circumference of a circle, centre O .

Angle $BOD = x^\circ$

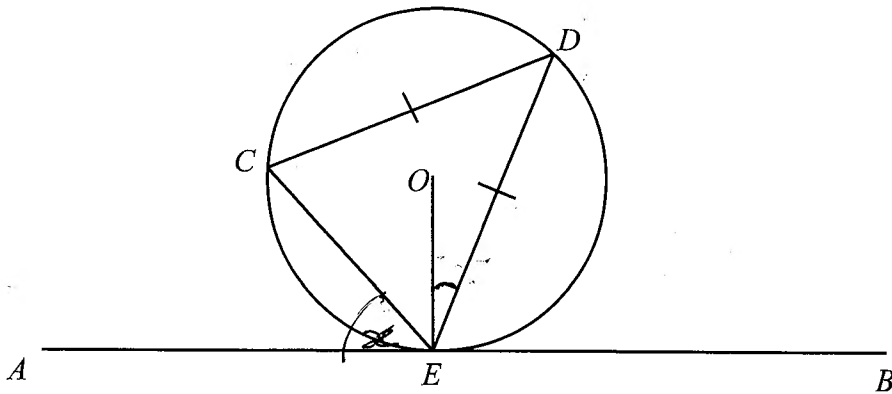
Find the size of angle BCD , in terms of x .

Give reasons for each stage of your working.

$$BAD = \frac{1}{2}x \quad \text{Angle at circumference is half angle at centre}$$

$$BCD = \underline{\underline{180 - \frac{1}{2}x}} \quad \text{opposite angles in a cyclic quadrilateral add to } 180^\circ$$

(Total for Question 17 is **3** marks)



C, D and E are points on a circle, centre O .
 AEB is a tangent to the circle at E .

$$CD = DE$$

$$\text{Angle } AEC = x^\circ$$

Find the size of angle OED , in terms of x .
 Give reasons for each stage of your working.

$$CDE = x \quad \text{Alternate segment theorem}$$

$$CED = \frac{180 - x}{2} \quad \begin{array}{l} \text{Angles at the base} \\ \text{of an isosceles triangle} \\ \text{are equal} \end{array}$$

$$OEC = 90 - x \quad OEA = 90^\circ \quad \begin{array}{l} \text{Tangent meets} \\ \text{radius at } 90^\circ \end{array}$$

$$OED = \frac{180 - x}{2} - (90 - x)$$

$$= 90 - \frac{1}{2}x - 90 + x$$

$$= \underline{\underline{\frac{1}{2}x}}$$

(Total for Question 18 is 5 marks)